NONLINEAR AND LINEARIZED PROBLEMS OF PULSATING GAS MOVEMENT IN A PIPELINE

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UDC 533.697

The numerical solution of a system of equations of one-dimensional isothermal movement of an ideal gas in a pipe with periodic variation in the stream velocity at the boundary is compared with the analytical solution of the linearized problem. The regions of variation in the parameters in which linearization provides satisfactory agreement are distinguished. The results obtained can be used in the analysis and modeling of pulsating gas movement in the pipelines of piston compressors [1].

The calculation of the one-dimensional isothermal movement of an ideal gas in a horizontal cylindrical pipe comes down to the solution of a system of quasilinear hyperbolic equations [2]

$$\begin{gathered} -\frac{\partial P}{\partial t}+\frac{\partial}{\partial x}(PW)=0 \\ \frac{\partial}{\partial t}(PW)+C^2\frac{\partial P}{\partial x}+\frac{\partial}{\partial x}(PW^2)+\frac{\lambda}{2P}PW\,|W|=0 \\ 0<x<L, \quad t>0 \end{gathered} \tag{1}$$

Here x is the coordinate along the pipe axis, t is the time, W and P are the gas velocity and pressure averaged over a cross section, C is the speed of sound, L and D are the length and diameters of the pipe, and λ is the coefficient of friction.

The initial and boundary conditions for system (1) are assigned in the form

$$\begin{gathered} W(x,0)=W_0=\text{const}, \quad P(x,0)=P_0\left[1+\frac{\lambda}{2D}\frac{W_0^2}{C^2}(L-x)\right] \\ W(0,t)=W_0+W_*\sin\omega t, \qquad P(L,t)=P_0=\text{const} \end{gathered}$$

The system (1), (2)is reduced to dimensionless form using the substitutions

$$\begin{gathered} \xi=\frac{x}{L}, \ \tau=\frac{Ct}{L}, \ p=\frac{P}{P_0}, \ w=\frac{W}{W}, \ V=\frac{W_*}{W_0}, \ M=\frac{W_0}{C} \\ H=\frac{\omega L}{C}, \qquad R=\frac{\lambda}{2}\frac{L}{D} \end{gathered}$$

In the analysis of pulsating gas movement in a pipeline instead of the nonlinear problem (1), (2) one usually considers the linearized problem [1, 2]

$$-\frac{\partial p}{\partial \xi}=\frac{\partial w}{\partial \tau}+2RM^2w, \qquad -\frac{\partial p}{\partial \tau}=M\frac{\partial w}{\partial \xi} \tag{3}$$

$$\begin{gathered} w=(0,\tau)=1+V\sin H\tau, \qquad p(1,\tau)=1, \qquad w(\xi,0)=1 \\ p(\xi,0)=1+RM^2(1-\xi) \end{gathered} \tag{4}$$

The system (3)is obtained from (2) on the assumption that the gas velocity is much less than sonic velocity and the gas pressure and velocity can be represented in the form of a sum of stationary and small nonstationary components [3]. A comparison of the numerical solution of problem (1), (2) with the analytical solution of (3), (4), with different values of V, H, and R makes it possible to distinguish those regions of variation of the parameters in which linearization leads to satisfactory results.

For the solution of the nonlinear problem (1), (2) we used an implicit finite-difference system similar to the system used earlier in [4] for the solution of systems of quasilinear hyperbolic equations and in

Moscow. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 4, pp. 153-155, July-August, 1974. Original article submitted Jnuary 3, 1974.

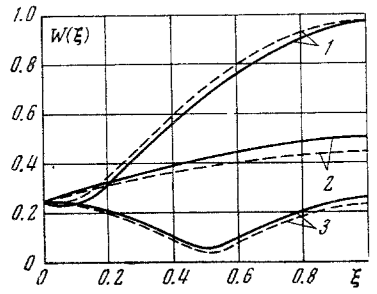

Fig. 1

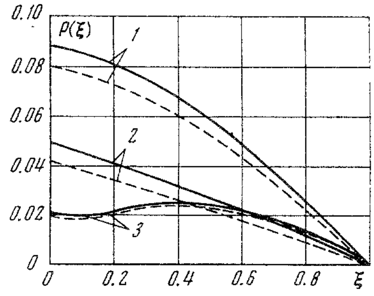

Fig. 2

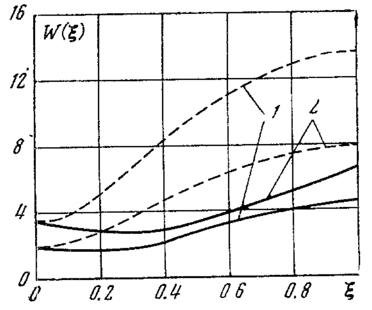

Fig. 3

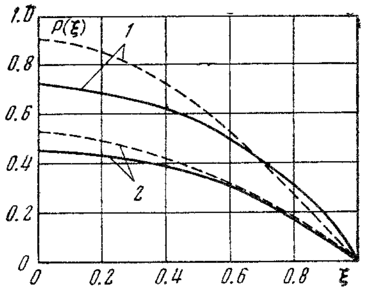

Fig. 4

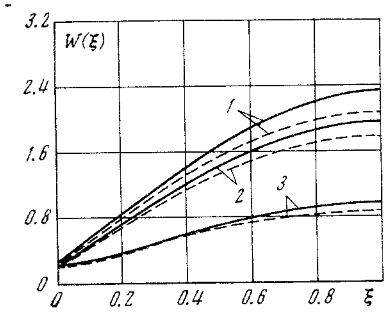

Fig. 5

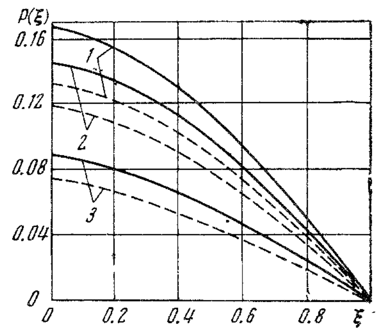

Fig. 6

application to the calculation of pulsating gas movement in a pipeline described in [5, 6]; the steps of the rectangular grid with respect to τ and ξ were 0.0015 and 0.05, respectively, and the numerical method was carried out on a BESM-6 computer.

The solution of the linearized problem (3), (4) has the form

$$w(\xi,\tau) = 1 + V\,\mathrm{Im}\left[\frac{\cos k(1-\xi)}{\cos k}\exp(jH\tau)\right] + \exp(-MR\tau)\sum_{n=0}^{\infty}[A_n\cos\lambda_n\tau + B_n\sin\lambda_n\tau]\sin\lambda_n\xi$$

$$p(\xi,\tau) = 1 + RM^2(1-\xi) + \frac{MV}{H}\,\mathrm{Re}\left[k\frac{\sin k(1-\xi)}{\cos k}\exp(jH\tau)\right]\exp(-MR\tau)\sum_{n=0}^{\infty}[A_n^*\cos\lambda_n\tau + B_n^*\sin\lambda_n\tau]\cos\lambda_n\xi \quad (5)$$

$$k = \alpha + j\beta = \sqrt{H^2 - 2jHMR}, \qquad \lambda_n = \frac{\pi}{2}(2n+1)$$

$$A_n = \frac{2\beta V(\alpha^2 + \lambda_n^2)}{[\beta^2 + (\alpha-\lambda_n)^2][\beta^2 + (\alpha+\lambda_n)^2]}$$

The other Fourier coefficients B_n, A_n^*, and B_n^* have an analogous structure and decrease just as fast with an increase in n.

In the calculations conducted for the 32 variants which encompass the limits of variation of the characteristics of a pulsating gas stream which are possible in practice [1], the following values of the parameters were taken: L = 20 m, C = 315 m/sec, $P_0 = 4.9 \cdot 10^5$ N/m^2, W_0 = 20 m/sec, λ = 0.02, and the remaining values were varied within the limits: V = 0.05-3.50, H = 1.05-3.14, R = 0.8-40.0.

A comparison of the numerical solution of the nonlinear problem (1), (2) and the equations (5) showed that good agreement in the frequency and phase of the oscillations is observed in the entire range of variation of the parameters. Therefore, a comparison was made with respect to the maximum deviation δ in the distribution over the length of the pipe of the amplitudes of the established oscillatory process.

For small amplitudes in the initial cross section (V = 0.25) δ does not exceed 16% in the entire range of frequencies, and the greatest error is observed at H = 1.05. In the region of higher frequencies (H = 1.57, 2.10, 2.62, 3.14) the amplitude distributions along the length almost coincide. The distributions along the length of the velocity and pressure amplitudes for the nonlinear and linearized problems are presented in Figs. 1 and 2 with solid and dashed curves, respectively, for V = 0.25, R = 4.0, and H = 1.57, 1.05, and 3.14 for curves 1-3, respectively.

With a fixed frequency H and an increase in V the value of δ increases, with the most significant deviations (over 100% at V = 3.50) being observed near the resonance frequency (H = 1.57). With departure from resonance (H = 2.10 and 2.62) the deviation δ is on the order of 10-15% in the entire range of variation of V.

An analysis of the data obtained shows that near the resonance frequency for all the values of V examined the deviation δ_w in the velocity amplitudes exceeds the deviation δ in the pressure amplitude by 1.5-2 times, while in nonresonance modes the values of δ_w and δ_p are close. As a rule, Eq. (5) gives an overstated velocity amplitude and an understated pressure compared with the numerical solution. The amplitude distributions for H = 1.57, R = 4.0, and V = 3.5 and 2.0 (curves 1 and 2, respectively) are presented in Figs. 3 and 4.

The dependence of the deviation δ on the parameter R characterizing the dissipative properties of the oscillatory system was studied near resonance (H = 1.46). The amplitude distributions of the gas velocity and pressure oscillations for H = 1.46, V = 0.25, and R = 0.8, 1.33, and 4.0 (curves 1-3) are shown in Figs. 5 and 6. With a decrease in R the deviation δ increases, reaching the order of 20% at R = 0.8, with the use of the linearized system (3), (4) leading to understated amplitudes.

Thus, far from the resonance frequencies in the entire range of variation in the parameters investigated the use of the linearized system (3), (4) leads to results which are in satisfactory agreement with the numerical solution of the nonlinear system (1), (2). If the frequency of the forced oscillations is close to the natural frequency of the gas column then the linearized model (3), (4) is not accurate enough when $V > 1$.

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